6/16/18 CWHMC

Text: Introduction to Number Theory (AoPS)

Chapter 1: Integers: The Basics

- The integers are defined as the set …-3, -2, -1, 0, 1, 2, 3…
    - Which of the following are integers: -4, 2, 1.66, 3, pi, sqrt(2)
- Divisibility:
    - We say m divides n if there exists a unique integer x such that n = mx. We write this as m | n.
    - Exercise:
        - Does 2 divide 4? 2 divide 5? 3 divide 12? 3 divide 14? -10 divide 30? 9 divide -36?
    - Does 0 divide any integer? Justify using the definition of divisibility.
    - If d is an integer other than 0, the following are equivalent:
        - n is a multiple of d
        - n is divisible by d
        - d divides n
    - Prove the above using the definition of divisibility.
    - You might notice that for any positive integer n, n **always** divides n. We can then define the term "proper divisor," which is a number dividing n that is not equal to n.

Chapter 2: Primes and Composites

- Prime numbers are defined as integers that have no divisors other than 1 and themselves. What's another way to define prime numbers using proper divisors?
    - **Warning:** 1 is not a prime number under this definition
    - Which of the following are primes: 2, 4, 8, 7, 12, 17
    - For contest math problems it is good to know the first few primes 2, 3, 5, 7… up to 50.

- Composite numbers are defined as integers that have more than 1 proper divisor. An important theorem in number theory is that every composite number can be broken down into the product of primes, known as the prime factorization. In fact, this factorization is unique!
    - Which of the following are composite: 19, 20, 21, 22, 23, 24, 25, 26, 27
    - For each number, write down its prime factorization. (Why do we not include 1 in the prime factorization of a number?)
- **Good to think about**: Why does such a factorization exist? And why is it unique?
- How do we determine if a number is prime? Specifically, for an integer n, must we test all prime divisors less than or equal to n?